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SAT/PSAT Quantitative

The SAT and PSAT Quantitative sections are designed to test your ability to reason mathematically, to understand basic math terminology, and to recall basic mathematic formulas and principles. You should be able to solve problems and apply relevant mathematics concepts in arithmetic, algebra, geometry, and data analysis.

EXAM✓PREP
StudyTIP

Be sure to practice the strategies and techniques covered in this chapter on the simulated tests found in Part IV of this book.

The SAT and PSAT Quantitative sections include multiple choice questions, each with five answer choices (A–E), as well as student-produced response questions, for which you are not provided any answer choices. All questions cover the content discussed in Chapter 7, "SAT/PSAT Math Review."

Multiple Choice

As we just mentioned, each multiple choice question includes five answer choices (A–E). The answer choices correspond to the circles on your answer sheet. You can use an approved calculator to assist you in answering any of the multiple choice questions, but none of the questions actually require the use of a calculator.

NOTE

Please visit www.sat.org for a list of approved calculators. Generally, calculators with memory and print functions are not allowed. The list of approved calculators is periodically updated, so check the website a couple of weeks before your test.

You will not receive credit for anything that you write in your test booklet, but you should work through the problems in the available space so that you can check your work. Be sure to do enough practice to determine just how much space you need to solve various problems. You can use whatever space is available in the section on which you are working, but you cannot move to another section in search of blank space to solve your math problems.

As mentioned earlier, if you don't know the answer to a question, mark it in your test booklet and come back to it later if you have time. If you are able to eliminate answer choices, cross them off in your test booklet. Make an educated guess if you are able to eliminate at least one answer choice. Remember that you get one point for each correct answer and zero points for answers that are left blank. If you answer a question incorrectly, you lose an additional fraction of a point.

Multiple choice questions will include answer choices in the Roman numeral format. Always take each Roman numeral as a true or false statement: Does it answer the question or not? As you evaluate each of the Roman numerals, eliminate answer choices based on whether the answer choices include the Roman numeral. This process might allow you to arrive at the correct answer without looking at every Roman numeral statement.

Consider the following example:

1. If x and y are both positive even integers, which of the following must be even?

  I. x^y

  II. $(x + 1)^y$

  III. $x^{(y + 1)}$

  (A) I only

  (B) II only

  (C) I and II only

  (D) I and III only

  (E) II and III only

Anatomy of an SAT and PSAT Multiple Choice Quantitative Question

Before we get to the strategies, we want you to understand what a Multiple Choice question looks like. Consider the following:

Question { If $5x - 6 = 14$, then $8x =$

  (A) $\frac{8}{5}$

  (B) 4

  (C) $\frac{64}{5}$

  (D) 20

  (E) 32
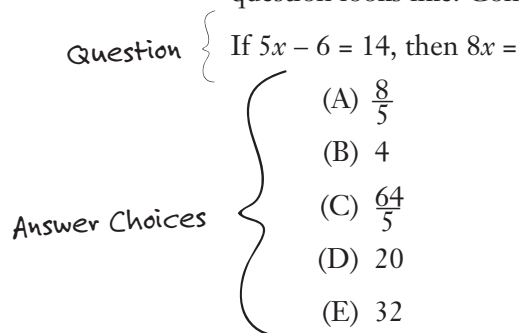
General Strategies for SAT and PSAT Multiple Choice Quantitative Questions

Remember these general strategies when approaching SAT/PSAT *quantitative*, or math, questions.

Draw Pictures

It really helps sometimes to visualize the problem. This strategy should not take a lot of time and can prevent careless errors. Sometimes you are given a figure or a table that you can work with; sometimes you just have to make your own. Consider the following example:

> The greatest number of diagonals that can be drawn from one vertex of a regular eight-sided polygon is
>
> (A) 1
>
> (B) 2
>
> (C) 3
>
> (D) 4
>
> (E) 5

The correct answer is E. To solve this problem, draw a diagram like the one that follows:
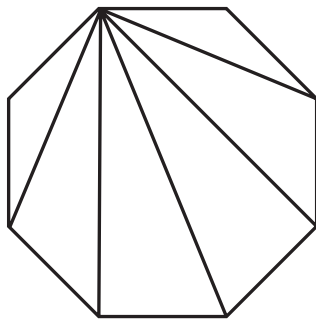
As you can see, if you draw an octagon (eight-sided polygon), you can only make five diagonals from one vertex.

Apply Logic

Even though you can use a calculator, most of the actual calculations are fairly simple. In fact, the SAT test writers are just as likely to test your logical reasoning ability or your ability to follow directions as they are to test your ability to plug numbers into an equation. Consider the following example:

If $b - c = 2$, and $a + c = 16$, then $a + b =$

    (A) 8

    (B) 14

    (C) 16

    (D) 18

    (E) 32

The correct answer is D. To solve this problem, first recognize that $(b - c) + (a + c) = a + b$. This is true because the c values cancel each other out, leaving you with $b + a$, which is equivalent to $a + b$. Therefore, $a + b$ must equal 2 + 16, or 18.

Answer the Question That You Are Asked

If the problem requires three steps to reach a solution and you completed only two of the steps, it is likely that the answer you arrived at will be one of the choices. However, it will not be the correct choice! Consider the following example:

6 ft

12 ft
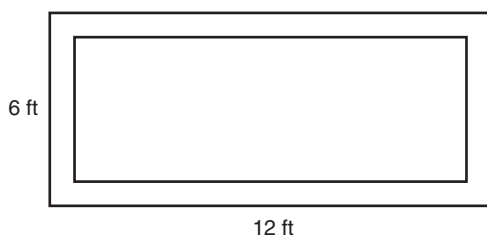
The rectangular garden shown in the figure above has a stone border 2 feet in width on all sides. What is the area, in square feet, of that portion of the garden that excludes the border?

    (A) 4

    (B) 16

    (C) 40

    (D) 56

    (E) 72

The correct answer is B. This problem is asking for the area of the middle portion of the garden. To solve this problem, perform the following calculations, and remember that the border goes around the entire garden. First, subtract the border width from the length of the garden:

$12 - 2(2) = 8$

Next, subtract the border width from the width of the garden:

$6 - 2(2) = 2$

The area (length × width) of the portion of the garden that excludes the border is 8 × 2, or 16.

If you accounted for only the border along one length and one width of the garden, you would have gotten answer choice C. Answer choice D is the area of the border around the garden. Answer choice E is the area of the entire garden, including the stone border.

Don't Quit Early

Reason your way through the problem so that it makes sense. Keep in mind, though, that these questions do not usually involve intensive calculations or complicated manipulations. Consider the following example:

> If $0 < pr < 1$, then which of the following CANNOT be true?
>
> (A) $p < 0$ and $r < 0$
>
> (B) $p < -1$ and $r < 0$
>
> (C) $p < -1$ and $r < -1$
>
> (D) $p < 1$ and $r < 1$
>
> (E) $p < 1$ and $r > 0$

The correct answer is C. At first glance, you might think that you don't have enough information to solve this problem. However, if you recognize that pr must be a positive fraction because it lies between 0 and 1, you can work your way through the answer choices and eliminate those that could be true:

▸ Answer choice A: If both p and r were less than 0, their product would be positive. It's possible for pr to be a positive fraction because both p and r could be negative fractions, so eliminate answer choice A.

▸ Answer choice B: If p were -1 and r were also a negative number, their product would be positive. It's possible for pr to be a positive fraction because r could be a negative fraction, so eliminate answer choice B.

▸ Answer choice C: If both p and r were less than -1, then pr would be greater than 1. This statement cannot be true, and answer choice C is correct.

▸ Answer choice D: If both p and r were less than 1, their product could be positive. It's possible for pr to be a positive fraction because both p and r could be negative fractions, so eliminate answer choice D.

▸ Answer choice E: If p were less than 1, p could be a positive fraction. If r were greater than 0, it would be a positive number, and it's possible for pr to be a positive fraction; eliminate answer choice E.

Check the Choices

Take a quick look at the answer choices as you read the problem for the first time. They can provide valuable clues about how to proceed. For example, many answer choices will be in either ascending or descending order. If the question asks you for the least possible value, try the smallest answer choice first. If it does not correctly answer the question, work through the rest of the answer choices from smallest to largest. Remember that one of them is the correct choice. Consider the following example:

If x is an integer and $y = 7x + 11$, what is the greatest value of x for which y is less than 50?

(A) 7

(B) 6

(C) 5

(D) 4

(E) 3

The correct answer is C. Because the question asks for the greatest value of x, start with answer choice A and use the process of elimination to find the correct answer.

▸ Answer choice A: $y = 7(7) + 11 = 60$. This is not less than 50, so eliminate answer choice A.

▸ Answer choice B: $y = 7(6) + 11 = 53$. This is not less than 50, so eliminate answer choice B.

▸ Answer choice C: $y = 7(5) + 11 = 46$. Because 5 is the greatest of the remaining answer choices and the result is less than 50, answer choice C must be correct.

Pick Numbers for the Variables

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Study**TIP**

When you pick numbers to substitute for the variables in a question, be sure to try both negative and positive numbers, unless the question indicates one or the other. For example, if the question states that $x < 5$, x could be a negative number.

You can sometimes simplify your work on a given problem by using actual numbers as "stand-ins" for variables. This strategy works when you have variables in the question and the same variables in the answer choices. You can simplify the answer choices by substituting actual numbers for the variables. Pick numbers that are easy to work with and that meet the parameters of the information given in the question. If you use this strategy, remember that numbers on the PSAT and SAT can be either positive or negative and are sometimes whole numbers and sometimes fractions. You should also be careful not to use 1 or 0 as your "stand-ins" because they can create "identities," which can lead to more than one seemingly correct answer choice.

In addition, it is sometimes necessary to try more than one number to see if the result always correctly responds to the question. If the numbers that you pick work for more than one answer choice, pick different numbers and try again, focusing on the remaining answer choices. Consider the following examples:

1. If x and y are both positive even integers, which of the following must be even?

   I. x^y

   II. $(x + 1)^y$

   III. $x^{(y + 1)}$

   (A) I only

   (B) II only

   (C) I and II only

   (D) I and III only

   (E) II and III only

The correct answer is D. The question states that both x and y are positive even integers. Therefore, you can pick any positive even integer and substitute that value for x and y in each of the Roman numeral choices, as follows:

▶ Roman Numeral I: $2^2 = 4$, which is even; $4^2 = 16$, which is also even. Any positive even integer raised to another positive even integer will result in an even number; therefore, Roman numeral I correctly answers the question. At this point, you could safely eliminate any answer choices that do not contain Roman Numeral I.

▶ Roman Numeral II: $(2 + 1)^2 = 3^2 = 9$, which is odd; $(4 + 1)^2 = 5^2 = 25$, which is also odd. When you add 1 to a positive even integer and raise the sum to a positive even integer, the result will be odd; therefore, Roman numeral II does not correctly answer the question. At this point, you could safely eliminate any remaining answer choices that contain Roman numeral II.

▶ Roman Numeral III: $2^{(2 + 1)} = 2^3 = 8$, which is even; $4^{(2 + 1)} = 4^3 = 64$, which is also even. Any positive even integer raised to an odd power will result in an even number; therefore, Roman numeral III correctly answers the question, and you can eliminate any remaining answer choices that do not contain Roman numeral III.

2. If a and b are positive consecutive odd integers, where $b > a$, which of the following is equal to $b^2 - a^2$?

    (A) $2a$

    (B) $4a$

    (C) $2a + 2$

    (D) $2a + 4$

    (E) $4a + 4$

The correct answer is E. You are given that both a and b are positive consecutive odd integers and that b is greater than a. Pick two numbers that fit the criteria: $a = 3$ and $b = 5$. Now, substitute these numbers into $b^2 - a^2$: $5^2 = 25$ and $3^2 = 9$; therefore, $b^2 - a^2 = 16$. Now, plug the value that you selected for a into the answer choices until one of them yields 16, as follows:

▶ $2(3) = 6$; eliminate answer choice A.

▶ $4(3) = 12$; eliminate answer choice B.

▶ $2(3) + 2 = 8$; eliminate answer choice C.

▶ $2(3) + 4 = 10$; eliminate answer choice D.

▶ $4(3) + 4 = 16$; answer choice E is correct.

Read the Questions Carefully

When you are looking at ratio problems, for example, note whether the question is giving a part-to-part ratio or a part-to-whole ratio, or both. The ratio of girls to boys in

a class is a part-to-part ratio. The ratio of girls to students in a class is a part-to-whole ratio. Consider the following example:

There are two types of candy in a bowl, chocolate and caramel. If the ratio of the number of pieces of chocolate candy to the total number of pieces of candy is $\frac{2}{3}$, each of the following could be the total number of pieces of candy, EXCEPT

(A) 6

(B) 12

(C) 15

(D) 20

(E) 30

The correct answer is D. To solve this equation, you must realize that if the number of total candies is expressed in thirds, the total number of candies must be divisible by 3. The number 20 does not break evenly into thirds. However, every other answer choice *does* break evenly into thirds. This is a part-to-whole ratio.

Student-Produced Response

One of the PSAT and SAT quantitative (math) sections includes 10 student-produced response, or grid-in, questions. These questions also cover the content discussed in Chapter 7. The only real difference between multiple choice questions and grid-in questions is that the latter do not include any answer choices. You must work out the problems in the space provided in your test booklet and fill in the circles on a special part of the answer sheet. You can use a calculator to assist you in answering any of the math questions, but none of the questions actually require the use of a calculator.

You will not receive credit for anything that you write in your test booklet, but you should work through the problems in the available space so that you can check your work. As mentioned earlier, if you don't know the answer to a question, mark it in your test booklet and come back to it later if you have time. Each correct answer is worth one point; you will not be penalized for marking an incorrect answer in this section, so it is to your advantage to fill in an answer, even if you're not sure it's correct. It is a good idea to practice with the answer sheet, or grid, on the next page. Tear it out and use it as you work through the student-produced response questions that follow. Only the answers that you actually put in the grid will be scored. The grid has four places and can accommodate only positive numbers and zero. As long as your answer is filled in completely, you can start in any column on the grid. The grid includes both decimal points and fraction lines, so you can grid your answer as either a decimal or a fraction. If your answer is zero, be sure to grid it in column 2, 3, or 4. Carefully read and understand the directions for this section (see Figure 6.1).

Directions: For Student-Produced Response questions 9-18, use the grids at the bottom of the answer sheet page on which you have answered questions 1-8.

Each of the remaining 10 questions requires you to solve the problem and enter your answer by marking the circles in the special grid, as shown in the examples below. You may use any available space for scratchwork.

Answer: $\frac{7}{12}$ Answer: 2.5   Answer: 201 Either position is correct.

Write answer in boxes. ⟶   ⟵ Fraction line

Grid in Result. ⟶

⟵ Decimal point

Note: You may start your answers in any column, space permitting. Columns not needed should be left blank.

• Mark no more than one circle in any column.

• Because the answer sheet will be machine-scored, **you will receive credit only if the circles are filled in correctly**.

• Although not required, it is suggested that you write your answer in the boxes at the top of the columns to help you fill in the circles accurately.

• Some problems may have more than one correct answer. In such cases, grid only one answer.

• No question has a negative answer.

• **Mixed numbers** such as $3\frac{1}{2}$ must be gridded as

3.5 or 7/2. (If $\boxed{3\,|\,1\,|\,/\,|\,2}$ is gridded, it will be

interpreted as $\frac{31}{2}$, not $3\frac{1}{2}$.)

Decimal Answers: If you obtain a decimal answer with more digits than the grid can accomodate, it may be either rounded or truncated, but it must fill the entire grid. enter the most accurate value the grid will accommodate. For example, if you obtain an answer such as 0.6666..., you should record your result as .666 or .667. **A less accurate value such as .66 or .67 will be scored as incorrect.**

Acceptable ways to grid $\frac{2}{3}$ are:

FIGURE 6.1 Directions for student-produced response questions.

Some student-produced response questions ask you for more than one answer. These questions might ask something such as, "What is one possible value of *x*?" You can fill in any one of the possible correct answers to the question.

General Strategies for SAT and PSAT Student Produced Response Questions

Most of the strategies that apply to the Multiple Choice questions work for the student-produced response questions, too. Use the following additional strategies when tackling more difficult student produced response question:

Manage the Grid

Remember that you will be using a special answer sheet for these questions. One important thing to keep in mind is that mixed numbers must be converted to improper fractions; for example, the computer cannot distinguish between $1\frac{2}{5}$ and $\frac{12}{5}$. Also, be sure that you have accurately recorded your answers on the grid, as only what appears on the answer sheet will be scored.

Double-Check your Answers

Because you do not have any answer choices to guide you, be very careful when performing the necessary calculations. Be sure to accurately set up the problems and write out each step so as not make any careless errors.

Consider the following examples:

1. A salesperson's monthly gross pay is $3,000 plus 30% of the dollar amount of her sales. If her gross pay for one month was $4,500, what was the dollar amount of her sales for that month?

The correct answer is 5,000. To solve this problem, first subtract the fixed amount ($3,000) from the gross pay ($4,500): $4,500 – $3,000 = $1,500. This amount is 30% of the dollar amount of her sales, so her sales must equal $1,500 ÷ .30, which is $5,000.

2. In the (*x*, *y*) coordinate system, the center of a circle has coordinates (3, 8), and the circle touches the *y*-axis at one point only. What is the diameter of the circle?

The correct answer is 6. If the center of the circle is at (3, 8) and the circle only touches the *y*-axis at one point, then it must touch at the point (0, 8). The radius of the circle is the change in the *x*-coordinate, or 3. Therefore, the diameter of the circle is 2(3), or 6.

3. Each of 11 people has a blank card on which they have written a positive integer. If the average (arithmetic mean) of these integers is 23, what is the greatest possible integer that could be on one of the cards?

The correct answer is 243. If the average of the 11 numbers is 23, then the sum of the 11 numbers must be 11 × 23, or 253. Say that 10 of the cards have a 1 written on them (the smallest positive integer); the largest possible number on the eleventh card, then, would be 253 – 10, or 243.

What's Next

If you require a review of basic math concepts, be sure to read Chapter 7, "Quantitative Review," before you tackle the practice exams in Part IV.